Final exam review
The following questions and problems are courtesy of Justin Dunlap

Practice Questions, June 5 Lecture

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Review question 1
A monochromatic beam of light of wavelength 600 nm is incident normally on a diffraction grating with a slit spacing of 1.70×10⁻⁴ cm. What is the angle for the first order maximum?

- [ ] A) 10.5°
- [ ] B) 12.1°
- [ ] C) 20.7°
- [ ] D) 16.4°
- [ ] E) 18.2°

\[ \sin \theta = \frac{\lambda}{d} \]
\[ \frac{0.6}{1.7} \]

Review question 2
A proton and an electron are both accelerated to the same final speed. If \( \lambda_p \) is the de Broglie wavelength of the proton and \( \lambda_e \) is the de Broglie wavelength of the electron, then

- [ ] A) \( \lambda_p > \lambda_e \)
- [ ] B) \( \lambda_p = \lambda_e \)
- [ ] C) \( \lambda_p < \lambda_e \)
- [ ] D) Not enough data to answer this question
- [ ] E) None of the above

\[ \lambda = \frac{h}{p} \]

Review question 3
When a beam of light, which is traveling in glass, strikes an air boundary, there is

- [ ] A) a 60° phase change in the reflected beam.
- [ ] B) a 45° phase change in the reflected beam.
- [ ] C) a 180° phase change in the reflected beam.
- [ ] D) no phase change in the reflected beam.
- [ ] E) a 90° phase change in the reflected beam.

Review question 4
A certain astronomical telescope has a diameter of 5.60 m. What is the minimum angle of resolution for this telescope at a wavelength of 620 nm?

- A) $3.11 \times 10^{-7}$ rad
- B) $2.70 \times 10^{-7}$ rad
- C) $1.35 \times 10^{-7}$ rad
- D) $1.11 \times 10^{-7}$ rad
- E) $4.05 \times 10^{-7}$ rad

Review question 5

Two spaceships approach Earth from the same direction. One has a speed of $0.21c$ and the other a speed of $0.34c$, both relative to Earth. What is the speed of one spaceship relative to the other?

- A) $0.16c$
- B) $0.13c$
- C) $0.18c$
- D) $0.15c$
- E) $0.14c$

Review question 6

Light of wavelength 687 nm is incident on a single slit 0.75 mm wide. At what distance from the slit should a screen be placed if the second dark fringe in the diffraction pattern is to be 1.7 mm from the center of the screen?

- A) 1.9 m
- B) 0.93 m
- C) 1.1 m
- D) 0.39 m
- E) 1.5 m

Review question 7

You are moving at a speed $23c$ relative to Randy, and Randy shines a light toward you. At what speed do you see the light passing you by?

- A) same, speed of light
- B) 13c
- C) 43c
- D) 23c
Review question 8

The first atomic explosion released approximately 1.0 \times 10^{14} J of energy. How much matter had to be changed into energy to produce this yield?

- [ ] A) 23 g
- [x] B) 1.1 g
- [ ] C) 13 g
- [ ] D) 0.45 kg
- [ ] E) 1.1 kg

Review question 9

The frequency of a light beam is doubled. Which one of the following is correct for the momentum of the photons in that beam of light?

- [ ] A) It stays the same.
- [ ] B) It is quadrupled.
- [ ] C) It is halved.
- [ ] D) It is reduced by one-fourth.
- [x] E) It is doubled.

Review question 10

The wavelength of light in a thin film is 360 nm and the wavelength of the same light in vacuum is 469 nm. What is the index of refraction for the film?

- [x] A) 1.30
- [ ] B) 1.10
- [ ] C) 1.70
- [ ] D) 1.50
- [ ] E) 1.90

Review question 11

What mass would have to be condensed to a radius of 1.0 \times 10^{-15} m (the order of magnitude of the radius of an atomic nucleus) in order for it to become a black hole? The Gravitational constant is \( G = 6.67384 \times 10^{-11} \text{N} \cdot \text{m}^2/\text{kg}^2 \).
Review question 12

In a two-slit experiment, the slit separation is $3.00 \times 10^{-5}$ m. The interference pattern is created on a screen that is $2.00$ m away from the slits. If the 7th bright fringe on the screen is a linear distance of $10.0$ cm away from the central fringe, what is the wavelength of the light?

- A) $224$ nm
- B) $204$ nm
- C) $234$ nm
- D) $214$ nm
- E) $100$ nm

\[
\sin \theta = \frac{d \sin \phi}{\lambda} = \frac{0.05}{0.2} = 0.04995 \\
\lambda = \frac{d \sin \phi}{\sin \theta} = \frac{30 \mu m}{0.05} = 15 \mu m = 0.015 \text{ mm}
\]

Review question 13

The surface temperature of the star is $6000$ K. What is the wavelength associated with the light emitted by this star?

- A) $907$ nm
- B) $492$ nm
- C) $850$ nm
- D) $502$ nm
- E) $311$ nm

\[
f_{peak} = \left( 5.8 \times 10^{10} \right) \frac{T}{6000} = 5.8 \times 10^{10} \times \frac{6000}{6000} = 3.5 \times 10^{14} \text{ Hz}
\]

\[
\lambda (f_{peak}) = \frac{c}{f} = \frac{3 \times 10^8}{3.5 \times 10^{14}} = 850 \text{ nm}
\]

Review question 14

An electron is moving with the speed of $1780$ m/s. What is its de Broglie wavelength?

- A) $502$ nm
- B) $302$ nm
- C) $205$ nm
- D) $420$ nm
- E) $409$ nm

\[
\lambda = \frac{h}{p} = \frac{6.6 \times 10^{-34}}{1.66 \times 10^{-27}} = 4 \times 10^{-7} \text{ m} = 400 \text{ nm}
\]
If the distance between the slits in Young's two-slit experiment is decreased, which one of the following statements is true of the interference pattern?

- [ ] A) The distance between the maxima decreases.
- [ ] B) The distance between the maxima stays the same.
- [ ] C) The distance between the minima increases.
- [ ] D) The distance between the minima stays the same.
- [ ] E) Impossible to tell without knowing the wavelength of light in use.

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**Practice Problems, June 5 Lecture**

*Solve three of the four problems (cross out the one you do not want graded). Show all of your work to receive full credit, most importantly show all the formulas you used to find the final answers. No credit will be awarded if an answer is given without work shown.*

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**Review problem 1**

You observe a sheet of a soap film ($n=1.33$) illuminated by a 500-nm-wavelength light on the same side of the film as you.

- a) If the beam of light is incident normal to the film, what is the minimum thickness of the film that will make it look bright? (4 pts)
  - ...
  - ...
- b) At this thickness, what is one other wavelength of light that would work to get a bright appearance? (4 pts)
  - ...
  - ...
- c) What is the next thickness that would make it look bright (using the original wavelength of light)? (4 pts)
  - ...
  - ...
- d) What would you observe at the top of the film where the soap is very thin? Why? (3 pts)
  - ...
  - ...

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**Review problem 2**

The [Large Hadron Collider](https://en.wikipedia.org/wiki/Large_Hadron_Collider) (LHC) at [CERN](https://en.wikipedia.org/wiki/CERN) in Europe can accelerate protons (rest mass, $m_p=1.673\times10^{-27}\text{kg}$, $c=2.998\times10^8\text{m/s}$) to a speed of $v=0.9999999991c$.  

$$v^2 = \left(1 - \frac{2\times10^{-9}}{c^2}\right)c^2$$
a) What is the rest energy (in MeV) of a proton? (1 eV = 1.602 × 10⁻¹⁹ J) (5 pts)
\[ \varepsilon = mc^2 \]

b) Find the magnitude of the relativistic momentum of the protons in the LHC. Comparing it with the rest nonrelativistic value, is it bigger or smaller? By what factor? (5 pts)

... ... ...

c) Compute a proton's total energy in the LHC (in GeV). How much more energy is this compared to the rest energy? (5 pts)

... ... ...

Review problem 3

Match the definitions and descriptions with the best term or phrase given below (1.5 pts each):

| Coherent light                          | Special Relativity | Quantum mechanics |
| Incoherent light                        | General Relativity | Blackbody radiation |
| Bright fringe                          | Principle of Equivalence | Wien's displacement law |
| Dark fringe                            | Proper length      | Plank's constant   |
| Monochromatic light                    | Proper time        | Quantized energy   |
| Superposition                          | Time dilation      | Photoelectric effect |
| Interference                           | Length contraction | Photon             |
| Young's Two-Slit Experiment            | Relativistic moment | Ultraviolet catastrophe |
| Phase change due to reflection         | Speed of light     | Work function      |
| Air wedge                              | Relativistic energy | Cutoff frequency |
| Newton's rings                         | Rest mass          | Rest mass of a photon |
| Thin film effects                      | Rest energy        | Photon scattering |
| Diffraction                            | Relativistic kinetic energy | Compton effect |
| Single slit diffraction                | Antimatter         | de Broglie wavelength |
| Resolution                             | Positron           | Wave-particle duality |
| Rayleigh's Criterion                   | Electron           | Heisenberg uncertainty principle |
| Diffraction grating                    | Proton             | Quantum tunneling |
| Reflection grating                     | Antiproton         | Schwarzschild radius |
Black hole

1. When a positron and this encounter each other, pure energy is given off in the form of gamma rays.
   - ...
2. In other words, all physical experiments conducted in a uniform gravitational field and in an accelerated frame of reference give identical results.
   - ...
3. Evidence suggesting matter has this quantity (S.I. unit: meter) is a principle of quantum mechanics.
   - ...
4. A constant that defines the smallest values, such as energy, a system in the universe can have.
   - ...
5. Light striking a metal ultimately resulting in a measurable current.
   - ...
6. A problem with classical physics suggesting that a heated object would release large amounts of high frequency light.
   - ...
7. Light is bounced off this surface made of many grooves, resulting in interference.
   - ...
8. Closer than this and the escape velocity would exceed that of light.
   - ...
9. The label given to the observation that if the energy of an event is known, how well you can measure time for that event is affected.
   - ...
10. A rainbow observed from a puddle in a parking lot is most likely due to this.
    - ...

Review problem 4

Kepler-62 (K-62) is a star 1,200.0 light years from Earth as measured by Earth astronomers. It appears there are at least two planets approximately the mass of the Earth in the habitable zone of K-62. You are tired of this planet and design a spaceship to get you to K-62.

1. What is the distance to K-62 in km as measured by Earth observers? (1 year =3.154×10^7s) (3 pts)
   - ...
2. Is the distance calculated in the previous part a proper length? Justify your answer. (3 pts)
   - ...
   - ...
3. How many years would pass according to you if you traveled to K-62 at a speed of v=0.999990c? (3 pts)
   - ...
4. According to you, what is the distance you traveled on your trip? (3 pts)

5. Say you can only get your spaceship up to 0.99900 c, how many more years will you age on your trip compared to the trip in part 3? (3 pts)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_0$ (BJD - 2454900)</td>
<td>103.9189 ± 0.0009</td>
<td>67.651 ± 0.0008</td>
<td>113.8117 ± 0.0003</td>
<td>83.404 ± 0.0003</td>
<td>522.710 ± 0.0006</td>
</tr>
<tr>
<td>$P$ (days)</td>
<td>5.714932 ± 0.000009</td>
<td>12.4417 ± 0.0001</td>
<td>18.16406 ± 0.000002</td>
<td>122.3874 ± 0.0008</td>
<td>267.291 ± 0.0005</td>
</tr>
<tr>
<td>Transit duration (hours)</td>
<td>2.31 ± 0.09</td>
<td>3.02 ± 0.09</td>
<td>2.97 ± 0.09</td>
<td>6.92 ± 0.16</td>
<td>7.46 ± 0.20</td>
</tr>
<tr>
<td>Depth (%)</td>
<td>0.043 ± 0.001</td>
<td>0.007 ± 0.001</td>
<td>0.092 ± 0.002</td>
<td>0.070 ± 0.003</td>
<td>0.042 ± 0.004</td>
</tr>
<tr>
<td>$R_p/R_*$</td>
<td>0.0188 ± 0.00003</td>
<td>0.0077 ± 0.0004</td>
<td>0.0278 ± 0.00006</td>
<td>0.0232 ± 0.0003</td>
<td>0.0203 ± 0.0008</td>
</tr>
<tr>
<td>$a/R_*$</td>
<td>18.7 ± 0.5</td>
<td>31.4 ± 0.8</td>
<td>40.4 ± 1.0</td>
<td>144 ± 4</td>
<td>243 ± 6</td>
</tr>
<tr>
<td>$b$</td>
<td>0.25 ± 0.13</td>
<td>0.16 ± 0.09</td>
<td>0.22 ± 0.13</td>
<td>0.06 ± 0.05</td>
<td>0.41 ± 0.14</td>
</tr>
<tr>
<td>$i$</td>
<td>89.2 ± 0.4</td>
<td>89.7 ± 0.2</td>
<td>89.7 ± 0.3</td>
<td>89.98 ± 0.02</td>
<td>89.90 ± 0.03</td>
</tr>
<tr>
<td>$e\cos \omega$</td>
<td>0.01 ± 0.17</td>
<td>−0.05 ± 0.14</td>
<td>−0.03 ± 0.24</td>
<td>0.05 ± 0.17</td>
<td>−0.05 ± 0.14</td>
</tr>
<tr>
<td>$e\sin \omega$</td>
<td>−0.07 ± 0.06</td>
<td>−0.18 ± 0.11</td>
<td>0.09 ± 0.09</td>
<td>−0.12 ± 0.02</td>
<td>−0.08 ± 0.10</td>
</tr>
<tr>
<td>$a$ (AU)</td>
<td>0.0553 ± 0.0005</td>
<td>0.0929 ± 0.0009</td>
<td>0.120 ± 0.001</td>
<td>0.427 ± 0.0004</td>
<td>0.718 ± 0.0007</td>
</tr>
<tr>
<td>$R_p(R_\odot)$</td>
<td>1.31 ± 0.04</td>
<td>0.54 ± 0.03</td>
<td>1.95 ± 0.07</td>
<td>1.61 ± 0.05</td>
<td>1.41 ± 0.07</td>
</tr>
<tr>
<td>$T_{eq}$ (K)</td>
<td>750 ± 41</td>
<td>578 ± 31</td>
<td>510 ± 28</td>
<td>270 ± 15</td>
<td>208 ± 11</td>
</tr>
</tbody>
</table>

A table you might want to take with you on the trip, but will not do you any good for this test.

Adapted from: W.J. Borucki et al., “Kepler-62: A Five-Planet System with Planets of 1.4 and 1.6 Earth Radii in
**Review Problem 1**

(a) \[ \lambda_{\text{film}} = \frac{\lambda_{\text{vac}}}{n} = \frac{500 \text{ nm}}{1.33} = \frac{500}{1.33} = 376 \text{ nm} \]

Round trip:

2. thickness = \[ \frac{\lambda_{\text{film}}}{2} = \frac{376}{2} = 188 \text{ nm} \]

(b)

2. thickness = \( \frac{3}{2} \lambda = 94 \times 3 = 282 \text{ nm} \)

\[ \frac{3}{4} \lambda = 3 \left( \frac{\lambda}{4} \right) = 282 \text{ nm} \]

Constructive interference:

\[ 2n(\text{thickness}) - 0.5 = m \]

\( \left( \frac{2n}{m} \right) t = m + 0.5 \)

\[ t = \frac{\lambda}{2n} (m + 0.5) \]

\( m = 0, 1, 2 \)

Dark Bright

Extra \( \pi \) (dissimilar reflection)

\( \lambda \) \[ \lambda = \frac{n}{z_n} (m-0.5) \]

No extra \( \pi \) similar

\( 2\pi \) extra

(c) 282 nm

Use vacuum

\( \lambda = \frac{n}{z_n} \)

Equation

\[ \frac{500}{2} = 164 \text{ nm} \]
(a) $v = c, E = mc^2$

\[
\frac{1.67 \times 10^{-24} \times 9 \times 10^{16}}{\frac{m^2}{ct}} = 1.5 \times 10^{-10}
\]

\[
\frac{1.5 \times 10^{-10} J}{1.6 \times 10^{-19} J/eV} = 9.4 \ldots 10^9 eV
\]

\[
0.94 \text{ GeV}
\]

\[
940 \text{ MeV}
\]

(b) $p = m_0 v \sqrt{1 - \frac{v^2}{c^2}} = \frac{1.6 \times 10^{-2} (3 \times 10^8)}{\sqrt{1 - (1 - 18 \times 10^{-9})}} = \frac{1}{\sqrt{18 \times 10^{-9}}} = 7.453
\]

\[
v^2 = (1 - 18 \times 10^{-9})c^2
\]

(c) $mc^2 = m_0 \sqrt{c^2} = (7451) \cdot 0.94 \text{ GeV} \approx 71000 \text{ GeV}
\]

\[
7451 \cdot 0.94 \text{ GeV} = 6996 \text{ GeV}
\]